

On the $\Delta\Delta$ component of the deuteron in the relativistic field theory model of the deuteron

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Abstract

The $\Delta\Delta$ component of the deuteron, where Δ stands for the $\Delta(1232)$ resonance, is calculated in the relativistic field theory model of the deuteron. For the probability of the $\Delta\Delta$ component of the deuteron we give $P(\Delta\Delta) = 0.08\%$. This prediction agrees good with the experimental estimate $P(\Delta\Delta) < 0.4\%$ at 90% of CL (D. Allasia *et al.*, Phys. Lett. B174 (1986) 450).

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1 Introduction

As has been stated in Ref.[1] that nowadays there is a consensus concerning the existence of non-nucleonic degrees of freedom in nuclei. The non-nucleonic degrees of freedom can be described either within QCD in terms of quarks and gluons [2] or in terms of mesons and nucleon resonances [3].

In this letter we investigate the non-nucleonic degrees of freedom in terms of the $\Delta(1232)$ resonance and calculate the contribution of the $\Delta\Delta$ component of the deuteron in the relativistic field theory model of the deuteron (RFMD) [4–10]. As has been shown in Ref.[7] the RFMD is motivated by QCD. The deuteron appears as a neutron–proton collective excitation, the Cooper np–pair, induced by a phenomenological local four–nucleon interaction in the nuclear phase of QCD. The RFMD describes the deuteron coupled to hadrons through one–nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon–loop anomalies which are completely determined by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies to effective Lagrangians of low–energy nuclear interactions is justified in the large N_C expansion, where N_C is the number of quark colours [7]. As has been shown in Refs.[8,9] the RFMD describes very good the processes of astrophysical interest such as the neutron–proton radiative capture $n + p \rightarrow D + \gamma$, disintegration of the deuteron by anti-neutrinos and neutrinos caused by $\bar{\nu}_e + D \rightarrow e^+ + n + n$ charged and $\bar{\nu}_e + D \rightarrow \bar{\nu}_e + n + p$ neutral weak currents [8], the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$ and the pep–process $p + e^- + p \rightarrow D + \nu_e$ [9].

The phenomenological npD interaction is given by [4–9] the phenomenological Lagrangian

$$\mathcal{L}_{\text{npD}}(x) = -ig_V[\bar{p}(x)\gamma^\mu n^c(x) - \bar{n}(x)\gamma^\mu p^c(x)]D_\mu(x), \quad (1.1)$$

where $D_\mu(x)$, $n(x)$ and $p(x)$ are the interpolating fields of the deuteron, the neutron and the proton [4–9]. The phenomenological coupling constant g_V is related to the electric quadrupole moment of the deuteron $Q_D = 0.286 \text{ fm}$: $g_V^2 = 2\pi^2 Q_D M_N^2$ [4,7], where $M_N = 940 \text{ MeV}$ is the nucleon mass. In the isotopically invariant form the phenomenological interaction Eq.(1.1) can be written as

$$\mathcal{L}_{\text{npD}}(x) = g_V \bar{N}(x)\gamma^\mu \tau_2 N^c(x) D_\mu(x), \quad (1.2)$$

where τ_2 is the Pauli isotopical matrix and $N(x)$ is a doublet of a nucleon field with components $N(x) = (p(x), n(x))$, $N^c(x) = C \bar{N}^T(x)$ and $\bar{N}^c(x) = N^T(x) C$, where C is a charge conjugation matrix and T is a transposition.

In the RFMD [5,8] the $\Delta(1232)$ resonance is the Rarita–Schwinger field [10] $\Delta_\mu^a(x)$, the isotopical index a runs over $a = 1, 2, 3$, having the following free Lagrangian [11,12]:

$$\mathcal{L}_{\text{kin}}^\Delta(x) = \bar{\Delta}_\mu^a(x)[-(i\gamma^\alpha \partial_\alpha - M_\Delta)g^{\mu\nu} + \frac{1}{4}\gamma^\mu \gamma^\beta (i\gamma^\alpha \partial_\alpha - M_\Delta)\gamma_\beta \gamma^\nu]\Delta_\nu^a(x), \quad (1.3)$$

where $M_\Delta = 1232 \text{ MeV}$ is the mass of the $\Delta(1232)$ resonance field $\Delta_\mu^a(x)$. In terms of the

eigenstates of the electric charge operator the fields $\Delta_\mu^a(x)$ are given by [11,12]

$$\Delta_\mu^1(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_\mu^{++}(x) - \Delta_\mu^0(x)/\sqrt{3} \\ \Delta_\mu^+(x)/\sqrt{3} - \Delta_\mu^-(x) \end{pmatrix}, \quad \Delta_\mu^2(x) = \frac{i}{\sqrt{2}} \begin{pmatrix} \Delta_\mu^{++}(x) + \Delta_\mu^0(x)/\sqrt{3} \\ \Delta_\mu^+(x)/\sqrt{3} + \Delta_\mu^-(x) \end{pmatrix}, \quad (1.4)$$

$$\Delta_\mu^3(x) = -\sqrt{\frac{2}{3}} \begin{pmatrix} \Delta_\mu^+(x) \\ \Delta_\mu^0(x) \end{pmatrix}.$$

The fields $\Delta_\mu^a(x)$ obey the subsidiary constraints: $\partial^\mu \Delta_\mu^a(x) = \gamma^\mu \Delta_\mu^a(x) = 0$ [10–12]. The Green function of the free Δ -field is determined by

$$\langle 0 | T(\Delta_\mu(x_1) \bar{\Delta}_\nu(x_2)) | 0 \rangle = -i S_{\mu\nu}(x_1 - x_2). \quad (1.5)$$

In the momentum representation $S_{\mu\nu}(x)$ reads [5,8,11,12]:

$$S_{\mu\nu}(p) = \frac{1}{M_\Delta - \hat{p}} \left(-g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{1}{3} \frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{M_\Delta} + \frac{2}{3} \frac{p_\mu p_\nu}{M_\Delta^2} \right). \quad (1.6)$$

The most general form of the $\pi N \Delta$ interaction compatible with the requirements of chiral symmetry reads [11]:

$$\begin{aligned} \mathcal{L}_{\pi N \Delta}(x) &= \frac{g_{\pi N \Delta}}{2M_N} \bar{\Delta}_\omega^a(x) \Theta^{\omega\varphi} N(x) \partial_\varphi \pi^a(x) + \text{h.c.} = \\ &= \frac{g_{\pi N \Delta}}{\sqrt{6}M_N} \left[\frac{1}{\sqrt{2}} \bar{\Delta}_\omega^+(x) \Theta^{\omega\varphi} n(x) \partial_\varphi \pi^+(x) - \frac{1}{\sqrt{2}} \bar{\Delta}_\omega^0(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^-(x) \right. \\ &\quad \left. - \bar{\Delta}_\omega^+(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^0(x) - \bar{\Delta}_\omega^0(x) \Theta^{\omega\varphi} p(x) \partial_\varphi \pi^0(x) + \dots \right], \end{aligned} \quad (1.7)$$

where $\pi^a(x)$ is the pion field with the components $\pi^1(x) = (\pi^-(x) + \pi^+(x))/\sqrt{2}$, $\pi^2(x) = (\pi^-(x) - \pi^+(x))/i\sqrt{2}$ and $\pi^3(x) = \pi^0(x)$. The tensor $\Theta^{\omega\varphi}$ is given in Ref.[11]: $\Theta^{\omega\varphi} = g^{\omega\varphi} - (Z + 1/2)\gamma^\omega \gamma^\varphi$, where the parameter Z is arbitrary. The parameter Z defines the $\pi N \Delta$ coupling off-mass shell of the $\Delta(1232)$ resonance. There is no consensus on the exact value of Z . From theoretical point of view $Z = 1/2$ is preferred [11]. Phenomenological studies give only the bound $|Z| \leq 1/2$ [13]. The empirical value of the coupling constant $g_{\pi N \Delta}$ relative to the coupling constant $g_{\pi N N}$ is $g_{\pi N \Delta} = 2.12 g_{\pi N N}$ [14]. As has been shown in Ref.[8] for the description of the neutron–proton radiative capture for thermal neutrons the parameter Z should be equal to $Z = 0.438$. That is very close to $Z = 1/2$.

For the subsequent calculations of the $\Delta\Delta$ component of the deuteron it is useful to have the Lagrangian of the $\pi N \Delta$ interaction taken in the equivalent form

$$\mathcal{L}_{\pi N \Delta}(x) = \frac{g_{\pi N \Delta}}{2M_N} \partial_\varphi \pi^a(x) \bar{N}^c(x) \Theta^{\varphi\omega} \Delta_\omega^a(x)^c + \text{h.c.}, \quad (1.8)$$

where $\Delta_\omega^a(x)^c = C \bar{\Delta}_\omega^a(x)^T$. Now we can proceed to the evaluation of the $\Delta\Delta$ component of the deuteron.

2 Effective $\Delta\Delta$ D interaction

In the RFMD the existence of the $\Delta\Delta$ component of the deuteron we can understand in terms of the coupling constant $g_{\Delta\Delta D}$ of the effective $\Delta\Delta$ D interaction.

In order to evaluate the Lagrangian of the effective $\Delta\Delta$ D interaction we have to obtain, first, the effective Lagrangian of the transition $N + N \rightarrow \Delta + \Delta$. In the RFMD this effective Lagrangian can be defined as follows [5,8]

$$\begin{aligned} \int d^4z \mathcal{L}_{\text{eff}}^{\text{NN} \rightarrow \Delta\Delta}(z) &= -\frac{g_{\pi\text{N}\Delta}^2}{8M_{\text{N}}^2} \iint d^4x_1 d^4x_2 [\bar{\Delta}_\alpha^a(x_1) \Theta^{\alpha\beta} N(x_1)] \\ &\quad \times \frac{\partial}{\partial x_1^\beta} \frac{\partial}{\partial x_1^\varphi} [\delta^{ab} \Delta(x_1 - x_2)] [\bar{N}^c(x_2) \Theta^{\varphi\omega} \Delta_\omega^b(x_2)^c]. \end{aligned} \quad (2.1)$$

Since the transferred momenta are much less than the mass of the π -mesons, according to the prescription of the RFMD the Green function of the π -mesons $\Delta(x_1 - x_2)$ should be replaced by the δ -function [4–9]: $\Delta(x_1 - x_2) = \delta^{(4)}(x_1 - x_2)/M_\pi^2$. This reduces the effective Lagrangian of the $N + N \rightarrow \Delta + \Delta$ transition to the form

$$\begin{aligned} \int d^4z \mathcal{L}_{\text{eff}}^{\text{NN} \rightarrow \Delta\Delta}(z) &= -\frac{g_{\pi\text{N}\Delta}^2}{8M_\pi^2 M_{\text{N}}^2} \iint d^4x_1 d^4x_2 [\bar{\Delta}_\alpha^a(x_1) \Theta^{\alpha\beta} N(x_1)] \\ &\quad \times \frac{\partial}{\partial x_1^\beta} \frac{\partial}{\partial x_1^\varphi} \delta^{(4)}(x_1 - x_2) [\bar{N}^c(x_2) \Theta^{\varphi\omega} \Delta_\omega^a(x_2)^c]. \end{aligned} \quad (2.2)$$

In terms of the Lagrangians of the npD interaction and the $N + N \rightarrow \Delta + \Delta$ transition the Lagrangian of the effective $\Delta\Delta$ D interaction can be defined by [8]

$$\begin{aligned} \int d^4x \mathcal{L}_{\text{eff}}^{\Delta\Delta\text{D}}(x) &= -i \frac{g_{\text{V}}}{M_\pi^2} \frac{g_{\pi\text{N}\Delta}^2}{4M_{\text{N}}^2} \int d^4x d^4x_1 d^4x_2 D_\mu(x) \\ &\quad [\bar{\Delta}_\alpha^a(x_1) \Theta^{\alpha\beta} S_F(x - x_1) \gamma^\mu \tau_2 S_F^c(x - x_2) \Theta^{\varphi\omega} \Delta_\omega^a(x)^c] \frac{\partial}{\partial x_1^\beta} \frac{\partial}{\partial x_1^\varphi} \delta^{(4)}(x_1 - x_2), \end{aligned} \quad (2.3)$$

where $S_F(x - x_1)$ and $S_F^c(x - x_2)$ are the Green functions of the free nucleon and anti-nucleon fields, respectively.

Such a definition of the contribution of the $\Delta\Delta$ component to the deuteron is in agreement with that given by Niephaus *et al.* [15] in the potential model approach (PMA).

In the large N_C expansion [7] the Lagrangian of the effective $\Delta\Delta$ D interaction reduces itself to the local form and reads

$$\mathcal{L}_{\text{eff}}^{\Delta\Delta\text{D}}(x) = \frac{1}{16\pi^2} \frac{g_{\text{V}}}{M_\pi^2} \frac{g_{\pi\text{N}\Delta}^2}{4M_{\text{N}}^2} [\bar{\Delta}_\alpha^a(x) J^{\alpha\mu\omega} \tau_2 \Delta_\omega^a(x)^c] D_\mu(x), \quad (2.4)$$

where the structure function $J^{\alpha\mu\omega}$ is given by the momentum integral

$$J^{\alpha\mu\omega} = \int \frac{d^4k}{\pi^2 i} \Theta^{\alpha\beta} k_\beta \frac{1}{M_{\text{N}} - \hat{k}} \gamma^\mu \frac{1}{M_{\text{N}} - \hat{k}} k_\varphi \Theta^{\varphi\omega}. \quad (2.5)$$

The evaluation of the momentum integral in leading order in the large N_C expansion [7] gives

$$J^{\alpha\mu\omega} = \frac{1}{6} \Lambda_{\text{D}}^3 M_{\text{N}} \Theta^{\alpha\mu\omega} = \frac{1}{6} \Lambda_{\text{D}}^3 M_{\text{N}} \Theta^{\alpha\beta} (\gamma^\mu g_{\beta\varphi} - g_\beta^\mu \gamma_\varphi - g_\varphi^\mu \gamma_\beta) \Theta^{\varphi\omega}, \quad (2.6)$$

where $\Lambda_{\text{D}} = 115.729 \text{ MeV}$ [7] is a cut-off restricting from above 3-momenta of the fluctuating nucleon fields forming the physical deuteron [4,7].

Thus the Lagrangian of the effective $\Delta\Delta$ D interaction reads

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\Delta\Delta\text{D}}(x) &= g_{\Delta\Delta\text{D}}[\bar{\Delta}_\alpha^a(x)\Theta^{\alpha\mu\omega}\tau_2\Delta_\omega^a(x)^c]D_\mu(x) = \\ &= -i g_{\Delta\Delta\text{D}}[\bar{\Delta}_\alpha^-(x)\Theta^{\alpha\mu\omega}\Delta_\omega^{++}(x)^c - \bar{\Delta}_\alpha^{++}(x)\Theta^{\alpha\mu\omega}\Delta_\omega^-(x)^c \\ &\quad + \bar{\Delta}_\alpha^+(x)\Theta^{\alpha\mu\omega}\Delta_\omega^0(x)^c - \bar{\Delta}_\alpha^0(x)\Theta^{\alpha\mu\omega}\Delta_\omega^+(x)^c]D_\mu(x),\end{aligned}\tag{2.7}$$

where the effective coupling constant $g_{\Delta\Delta\text{D}}$ is defined by

$$g_{\Delta\Delta\text{D}} = g_V \frac{g_{\pi\text{NN}}^2}{384\pi^2} \frac{g_{\pi\text{N}\Delta}^2}{g_{\pi\text{NN}}^2} \frac{\Lambda_D^3}{M_\pi^2 M_N}.\tag{2.8}$$

On-mass shell of the $\Delta(1232)$ resonance, i.e. in the case of the PMA [1,15], the contribution of the parameter Z vanishes and the effective $\Delta\Delta$ D interaction acquires the form

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\Delta\Delta\text{D}}(x) &= g_{\Delta\Delta\text{D}} g^{\alpha\beta}[\bar{\Delta}_\alpha^a(x)\gamma^\mu\tau_2\Delta_\beta^a(x)^c]D_\mu(x) = \\ &= -i g_{\Delta\Delta\text{D}} g^{\alpha\beta}[\bar{\Delta}_\alpha^-(x)\gamma^\mu\Delta_\beta^{++}(x)^c - \bar{\Delta}_\alpha^{++}(x)\gamma^\mu\Delta_\beta^-(x)^c \\ &\quad + \bar{\Delta}_\alpha^+(x)\gamma^\mu\Delta_\beta^0(x)^c - \bar{\Delta}_\alpha^0(x)\gamma^\mu\Delta_\beta^+(x)^c]D_\mu(x).\end{aligned}\tag{2.9}$$

The total probability of the existence of the $\Delta\Delta$ component in the deuteron we can estimate by the quantity

$$P(\Delta\Delta) = 2 \frac{g_{\Delta\Delta\text{D}}^2}{g_V^2} = 0.08\%.\tag{2.10}$$

Our theoretical prediction agrees good with the recent experimental estimate of the upper limit $P(\Delta\Delta) < 0.4\%$ at 90% of CL [16] quoted by Dymarz and Khanna [1].

3 Conclusion

The theoretical estimate of the $\Delta\Delta$ component of the deuteron obtained in the RFMD agrees good with the experimental upper limit. Indeed, for the $\Delta(1232)$ resonance on-mass shell [1,15] we predict $P(\Delta\Delta) = 0.08\%$ whereas experimentally $P(\Delta\Delta)$ is restricted by $P(\Delta\Delta) < 0.4\%$ at 90% of CL [16].

Off-mass shell of the $\Delta(1232)$ resonance, where the parameter Z should contribute, our prediction for $P(\Delta\Delta)$ can be changed, of course. Moreover, the contribution of the $\Delta\Delta$ component to different processes and quantities can be different. However, we would like to emphasize that in the RFMD by using the effective $\Delta\Delta$ D interaction determined by Eq.(2.7) one can calculate the contribution of the $\Delta\Delta$ component of the deuteron to any low-energy process with the deuteron in the initial or final state. These calculations are rather complicated and go beyond the scope of this letter.

In our approach we do not distinguish contributions of the $\Delta\Delta$ -pair with a definite orbital momentum $^3\text{S}_1^{\Delta\Delta}$, $^3\text{D}_1^{\Delta\Delta}$ and so on to the effective $\Delta\Delta$ D interaction Eq.(2.7). The obtained value of the probability $P(\Delta\Delta)$ should be considered as a sum of all possible states with a certain orbital momentum.

Our prediction $P(\Delta\Delta) = 0.08\%$ does not contradict to the predictions obtained in the PMA by Dymarz and Khanna [1]. In fact, as has been stated by Dymarz and Khanna

[1] their results agree with the experimental estimate of the upper limit of the total probability $P(\Delta\Delta)$ given by Allia *et al.* [16]. Unlike our approach Dymarz and Khanna have given a percentage of the probabilities of different states ${}^3S_1^{\Delta\Delta}$, ${}^3D_1^{\Delta\Delta}$ and so to the wave function of the deuteron. In our approach the deuteron couples to itself and other particles through the one-baryon loop exchanges. The Lagrangian of the effective $\Delta\Delta$ D interaction given by Eq.(2.7) defines completely the contribution of the $\Delta\Delta$ intermediate states to baryon-loop exchanges. The decomposition of the effective $\Delta\Delta$ D interaction according to the $\Delta\Delta$ states with a certain orbital momentum should violate Lorentz invariance for the evaluation of the contribution of every state. In the RFMD this can lead to incorrect results [7–9]. The relativistically covariant procedure of the decomposition of the interactions like the $\Delta\Delta$ D one in terms of the states with a certain orbital momenta is now in progress in the RFMD. However, a smallness of the contribution of the $\Delta\Delta$ component in the deuteron obtained in the RFMD makes such a decomposition applied to the $\Delta\Delta$ D interaction meaningless to some extent due to impossibility to measure the terms separately.

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